Eccentric hollow circular section

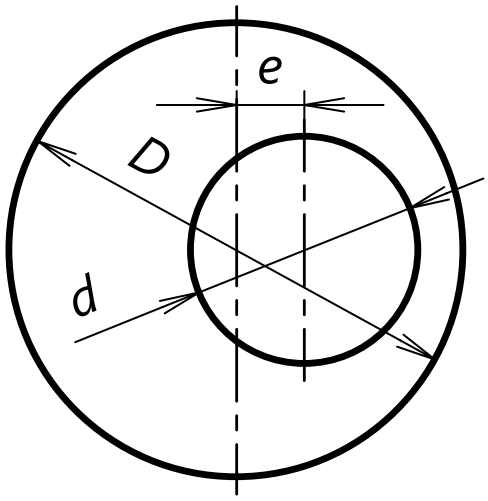

Fig. 1 - Eccentric hollow circular section

Values for calculation:

Twisting moment	T	1000	Nm
Outer diameter	D	100	mm
Inner diameter	d	50	mm
Eccentricity	e	20	mm
Length	L	1000	mm
Modulus of rigidity	G	80000	MPa

$$\lambda = \frac{e}{D} = \frac{20}{100} = 0.2$$

$$n = \frac{d}{D} = \frac{50}{100} = 0.5$$

$$C = 1 + \frac{16n^2}{(1-n^2)(1-n^4)}\lambda^2 + \frac{384n^4}{(1-n^2)^2(1-n^4)^4}\lambda^4$$

$$= 1 + \frac{16 \cdot 0.5^2}{(1-0.5^2)(1-0.5^4)}0.2^2 + \frac{384 \cdot 0.5^4}{(1-0.5^2)^2(1-0.5^4)^4}0.2^4 = 1.316$$

$$
\begin{aligned}
F &= 1 + \frac{4n^2}{1-n^2}\lambda + \frac{32n^2}{(1-n^2)(1-n^4)}\lambda^2 + \frac{48n^2\left(1+2n^2+3n^4+2n^6\right)}{(1-n^2)(1-n^4)(1-n^6)}\lambda^3 + \frac{64n^2\left(2+12n^2\right.}{(1-n} \\
&= 1 + \frac{4\cdot 0.5^2}{1-0.5^2}0.2 + \frac{32\cdot 0.5^2}{\left(1-0.5^2\right)\left(1-0.5^4\right)}0.2^2 + \frac{48\cdot 0.5^2\left(1+2\cdot 0.5^2+3\cdot 0.5^4+2\cdot 0.5^6\right.}{\left(1-0.5^2\right)\left(1-0.5^4\right)\left(1-0.5^6\right)} \\
&+ \frac{64\cdot 0.5^2\left(2+12\cdot 0.5^2+19\cdot 0.5^4+28\cdot 0.5^6+18\cdot 0.5^8+14\cdot 0.5^{10}+3\cdot 0.5^{12}\right)}{\left(1-0.5^2\right)\left(1-0.5^4\right)\left(1-0.5^6\right)\left(1-0.5^8\right)}0.2^4 \\
&= 2.209
\end{aligned}
$$

Polar moment of inertia:

$$K = \frac{\pi\left(D^4-d^4\right)}{32C} = \frac{\pi\left(100^4-50^4\right)}{32\cdot 1.316} = 6993833.4\text{mm}^4$$

Angle of twist:

$$\theta = \frac{T\cdot 10^3\cdot L}{KG} = \frac{1000\cdot 10^3\cdot 1000}{6993833.4\cdot 80000} = 0.002\text{rad}$$

Torsion stress:

$$\tau_{max} = \frac{16\cdot 10^3\cdot TDF}{\pi\left(D^4-d^4\right)} = \frac{16\cdot 10^3\cdot 1000\cdot 100\cdot 2.209}{\pi\left(100^4-50^4\right)} = 12\text{MPa}$$