Contact stress (sphere)

When two bodies with curved surfaces are pressed together, the line of contact changes to the contact surface and the stresses in the bodies become spatial. Contact stress problems occur at the point of contact of the wheel with the rail, in the valve manifolds of internal combustion engines between cams and valve tappets, in gear engagement and in rolling bearings. Characteristic disturbances that can be observed are cracks, wells or peeling of the surface layer of the material.
The most general example of contact stress occurs when each of the contacting bodies has two different radii of curvature-the radius in the rolling plane is different from the radius in the plane perpendicular thereto, both planes passing through the axes of thrust forces.

Contact stress:

$$\sigma_c = 0{,}918 \sqrt[3]{\frac{F}{K_D^2 C_E^2}} \leq \sigma_H$$

σ_c	contact stress	[MPa]
F	total force	[N]
K_D	dimensional coefficient	[mm]
C_E	material coefficient	[1/MPa]
σ_H	allowable hertz pressure	[MPa]

Material coefficient:

$$C_E = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

C_E	material coefficient	[1/MPa]
ν_1	Poisson's ratio 1	[]
ν_2	Poisson's ratio 2	[]
E_1	Young’s modulus 1	[MPa]
E_2	Young’s modulus 2	[MPa]

Allowable hertz pressure:

- for non-hardened material

$$\sigma_H = \frac{7HB}{S_F} * C_c$$

- for hardened material

$$\sigma_H = \frac{4{,}2R_{p0,2T}}{S_F} * C_c$$

σ_H	allowable hertz pressure	[MPa]
HB	hardness	[HB]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]

C_c	coefficient according to load	[]

Coefficient according to load:

load	[]
Static load	1
Unidirectional load, non-impact load	0,8
Unidirectional load, with a small impact load	0,7
Unidirectional load, with a big impact load	0,6
Alternating load, with a small impact load	0,45
Alternating load, with a big impact load	0,25

Diameter of circular contact area:

$$b = 1{,}442\,\sqrt[3]{F K_D C_E}$$

b	diameter of circular contact area	[mm]
F	total force	[N]
K_D	dimensional coefficient	[mm]
C_E	material coefficient	[1/MPa]

Contact stress two sphere:

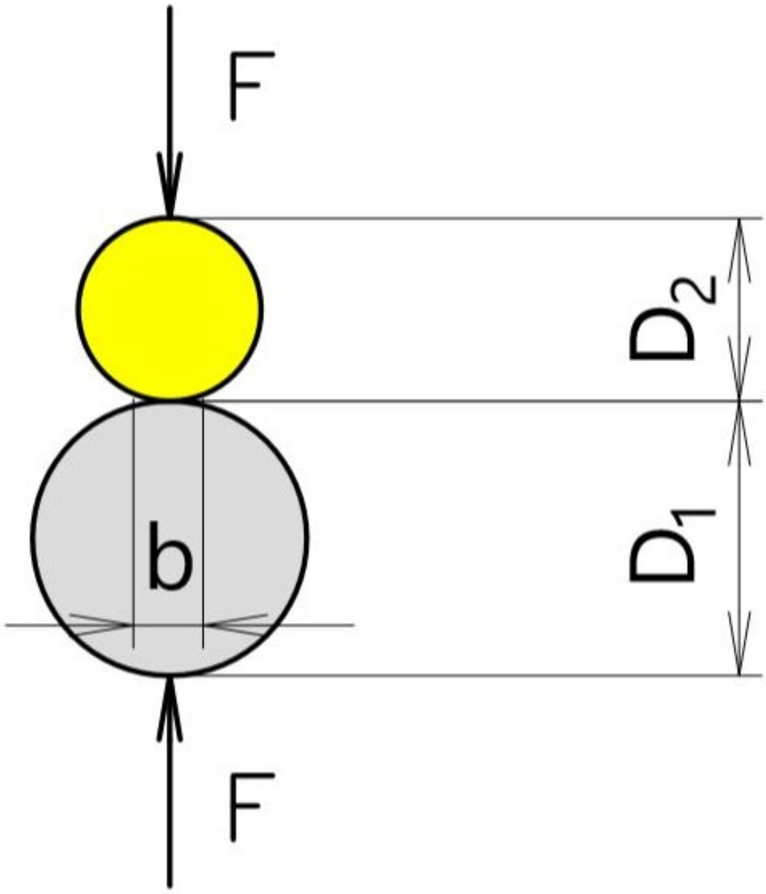

Fig. 1 contact stress two sphere

$$K_D = \frac{D_1 D_2}{D_1 + D_2}$$

K_D	dimensional coefficient	[mm]
D_1	diameter sphere 1	[mm]
D_2	diameter sphere 2	[mm]

Contact stress of sphere on flat surface:

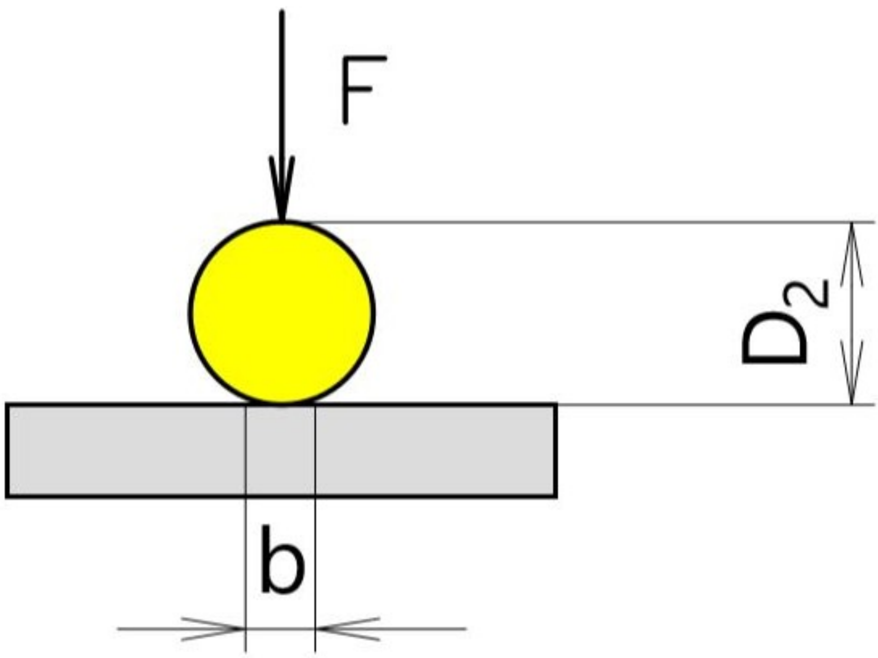

Fig. 2 contact stress of sphere on flat surface

$K_D = D_2$

K_D	dimensional coefficient	[mm]
D_2	diameter sphere	[mm]

Contact stress of the sphere in the spherical socket:

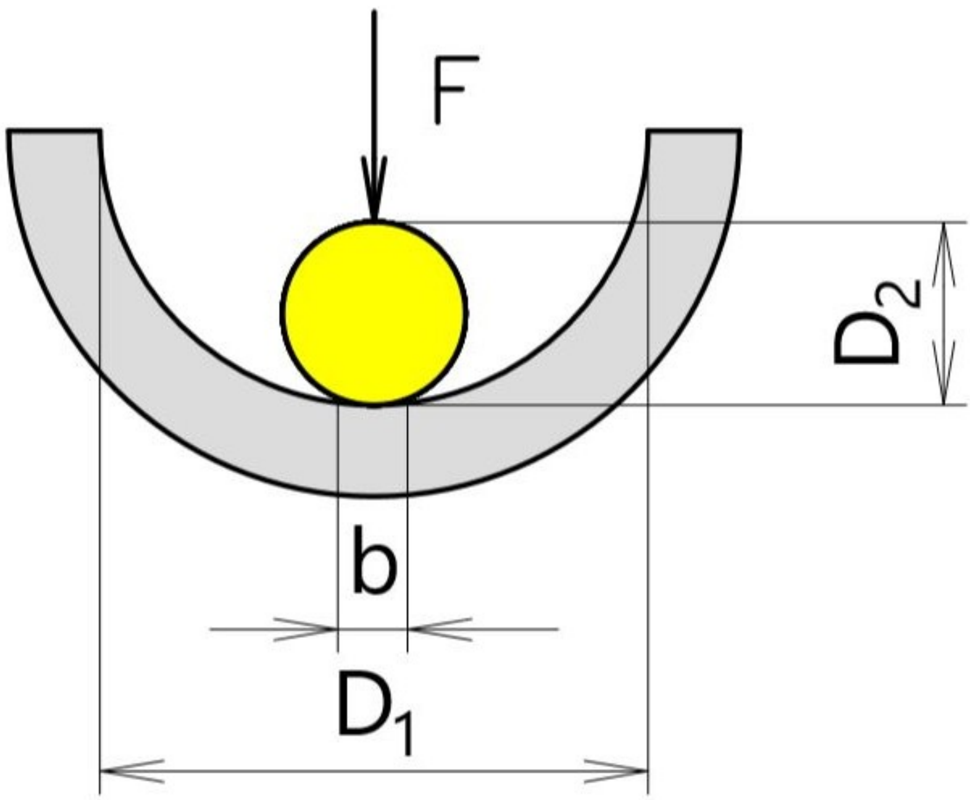

Fig. 3 contact stress of the sphere in the spherical socket

$$K_D = \frac{D_1 D_2}{D_1 - D_2}$$

K_D	dimensional coefficient	[mm]
D_1	diameter spherical socket	[mm]
D_2	diameter sphere	[mm]

Literature:

Warren C. Young, Richard G. Budynas: Roark's Formulas for Stress and Strain
ČSN EN 13001-3-3: Jeřáby – Návrh všeobecně – Část 3-3: Mezní stavy a prokázání způsobilosti kontaktů kolo/kolejnice.
Joseph E. Shigley, Charles R. Mischke, Richard G. Budynas: Konstruování strojních součástí 2010.