Radial pin for shaft-hub connection

The easiest and oldest way joints. It is a joint with a shape contact. The pin serves primarily to ensure the mutual positioning of the two parts. They are cylindrical or conical. The pins are dimensioned under simplified assumptions without will and without the pressing effect. When designing a pin, the cross section of the pin in the shear area must be the nominal cross-section of the pin.

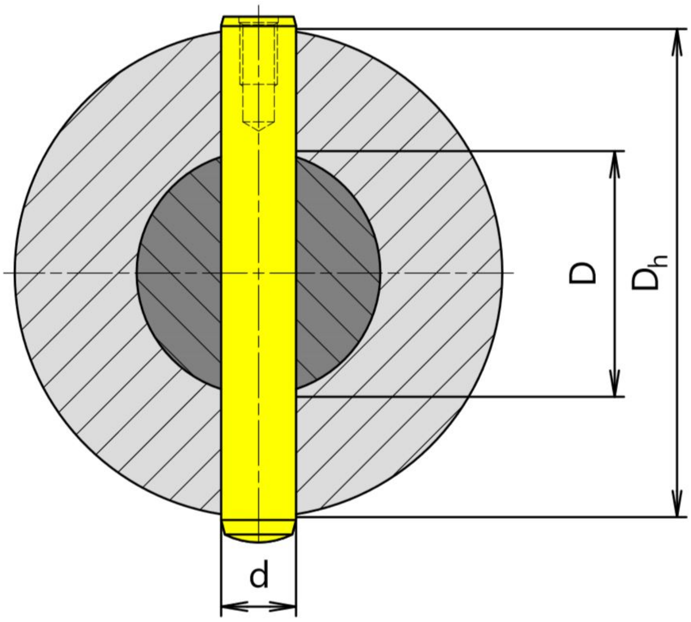

Fig. 1 Radial pin for shaft-hub connection

Torsion stress in the shaft:

$$\tau_s = \frac{16M_T}{\pi D^3} * \frac{J_{tube(s)}}{J_{net(s)}} \leq \tau_{all}$$

$$\frac{J_{tube(s)}}{J_{net(s)}} = 1/\frac{J_{net(s)}}{J_{tube(s)}}$$

τ_s	torsion stress in the shaft	[MPa]
M_T	torque	[Nm]
D	diameter of the shaft	[mm]
τ_{all}	allowable shear stress	[MPa]

Allowable shear stress:

$$\tau_{all} = \frac{0{,}4R_{p0,2T}}{S_F} * C_c$$

τ_{all}	allowable shear stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient according to load	[]

Coefficient according to load:

load	[]
Unidirectional load, non-impact load	0,8
Unidirectional load, with a small impact load	0,7
Unidirectional load, with a big impact load	0,6
Alternating load, with a small impact load	0,45
Alternating load, with a big impact load	0,25

Shear stress in the pin:

$$\tau_p = \frac{4M_T}{\pi * d^2 * D} + \frac{2F_A}{\pi * d^2} \leq \tau_{all}$$

τ_p	shear stress in the pin	[MPa]
M_T	torque	[Nm]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]
F_A	axial force	[N]
τ_{all}	allowable shear stress	[MPa]

Bearing stress in the pin and shaft:

$$p_1 = \frac{6M_T}{D^2 * d} + \frac{F_A}{D * d} \leq \sigma_{all}$$

P_1	bearing stress in the pin and shaft	[MPa]
M_T	torque	[Nm]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]
F_A	axial force	[N]
σ_{all}	allowable bearing stress	[MPa]

Allowable bearing stress:

$$\sigma_{all} = \frac{0{,}9R_{p0,2T}}{S_F} * C_c$$

σ_{all}	allowable bearing stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient according to load	[]

Bearing stress in the pin and hub:

$$p_2 = \frac{4M_T}{d * (D_h^2 - D^2)} + \frac{F_A}{d * (D_h - D)} \leq \sigma_{all}$$

P_2	bearing stress in the pin and hub	[MPa]
M_T	torque	[Nm]
D	diameter of the shaft	[mm]

d	diameter of the pin	[mm]
D_h	diameter of the hub	[mm]
F_A	axial force	[N]
σ_{all}	allowable bearing stress	[MPa]

Torsion stress in the hub:

$$\tau_h = \frac{16 M_T D_h}{\pi (D_h^4 - D^4)} * \frac{J_{tube(h)}}{J_{net(h)}} \leq \tau_{all}$$

$$\frac{J_{tube(h)}}{J_{net(h)}} = 1 / \frac{J_{net(h)}}{J_{tube(h)}}$$

τ_h	torsion stress in the hub	[MPa]
M_T	torque	[Nm]
D	diameter of the shaft	[mm]
D_h	diameter of the hub	[mm]
d	diameter of the pin	[mm]
τ_{all}	allowable shear stress	[MPa]

If the shaft and hub is loaded with the bending moment in the joint, the bending stress must be checked. If the shaft and hub is loaded with a shear force in the joint, the shear stress must be checked. The shaft and hub may be load in the joint by axial force. The shaft and hub must be checked for axial stresses. When calculating the different load types, it is necessary to calculate the combined stress.

Bending stress in the shaft:

$$\sigma_{B(s)} = \frac{32 M_B}{\pi D^3} * \frac{Z_{tube(s)}}{Z_{net(s)}} \leq \sigma_{Ball}$$

$$\frac{Z_{tube(s)}}{Z_{net(s)}} = 1 / \frac{Z_{net(s)}}{Z_{tube(s)}}$$

$\sigma_{B(s)}$	bending stress in the shaft	[MPa]
M_B	bending moment	[Nm]
D	diameter of the shaft	[mm]
σ_{Ball}	allowable bending stress	[MPa]

Allowable bending stress:

$$\sigma_{Ball} = \frac{0{,}6 R_{p0,2T}}{S_F} * C_c$$

σ_{Ball}	allowable bending stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient according to load	[]

Bending stress in the hub:

$$\sigma_{B(h)} = \frac{32 M_B D_h}{\pi(D_h^4 - D^4)} * \frac{Z_{tube(h)}}{Z_{net(h)}} \leq \sigma_{Ball}$$

$$\frac{Z_{tube(h)}}{Z_{net(h)}} = 1/\frac{Z_{net(h)}}{Z_{tube(h)}}$$

$\sigma_{B(h)}$	bending stress in the hub	[MPa]
M_B	bending moment	[Nm]
D	diameter of the shaft	[mm]
D_h	diameter of the hub	[mm]
σ_{Ball}	allowable bending stress	[MPa]

Shear stress in the shaft:

$$\tau_{s(s)} = \frac{4F_R}{\pi D^2} * \frac{A_{tube(s)}}{A_{net(s)}} \leq \tau_{all}$$

$$\frac{A_{tube(s)}}{A_{net(s)}} = 1/\frac{A_{net(s)}}{A_{tube(s)}}$$

$\tau_{s(s)}$	shear stress in the shaft	[MPa]
F_R	shear force	[N]
D	diameter of the shaft	[mm]
τ_{all}	allowable shear stress	[MPa]

Shear stress in the hub:

$$\tau_{s(h)} = \frac{4F_R}{\pi(D_h^2 - D^2)} * \frac{A_{tube(h)}}{A_{net(h)}} \leq \tau_{all}$$

$$\frac{A_{tube(h)}}{A_{net(h)}} = 1/\frac{A_{net(h)}}{A_{tube(h)}}$$

$\tau_{s(h)}$	shear stress in the hub	[MPa]
F_R	shear force	[N]
D	diameter of the shaft	[mm]
D_h	diameter of the hub	[mm]
τ_{all}	allowable shear stress	[MPa]

Axial stress in the shaft:

$$\sigma_{A(s)} = \frac{4F_A}{\pi D^2} * \frac{A_{tube(s)}}{A_{net(s)}} \leq \sigma_{Aall}$$

$$\frac{A_{tube(s)}}{A_{net(s)}} = 1/\frac{A_{net(s)}}{A_{tube(s)}}$$

$\sigma_{A(s)}$	axial stress in the shaft	[MPa]
F_A	axial force	[N]
D	diameter of the shaft	[mm]
σ_{Aall}	allowable axial stress	[MPa]

Allowable axial stress:

$$\sigma_{Aall} = \frac{0{,}45 R_{p0,2T}}{S_F} * C_c$$

σ_{Aall}	allowable axial stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient according to load	[]

Axial stress in the hub:

$$\sigma_{A(h)} = \frac{4F_A}{\pi(D_h^2 - D^2)} * \frac{A_{tube(h)}}{A_{net(h)}} \leq \sigma_{Aall}$$

$$\frac{A_{tube(h)}}{A_{net(h)}} = 1/\frac{A_{net(h)}}{A_{tube(h)}}$$

$\sigma_{A(h)}$	axial stress in the hub	[MPa]
F_A	axial force	[N]
D	diameter of the shaft	[mm]
D_h	diameter of the hub	[mm]
σ_{Aall}	allowable axial stress	[MPa]

Combined stress in the shaft:

$$\sigma_{tresca(s)} = \sqrt{\left(\frac{Z_{net(s)}}{Z_{tube(s)}} * K_{tB(s)} * \sigma_{B(s)}\right)^2 + \left(\frac{A_{net(s)}}{A_{tube(s)}} * K_{tA(s)} * \sigma_{A(s)}\right)^2 + 4\left(\left(\frac{J_{net(s)}}{J_{tube(s)}} * K_{ts} * \tau_s\right)^2 + \tau_{s(s)}^2\right)} \leq \sigma_{Call}$$

$$\frac{Z_{net(s)}}{Z_{tube(s)}} = 1 - (16/3/\pi)(d/D)$$

$$\frac{A_{net(s)}}{A_{tube(s)}} = 1 - 4/\pi(d/D)$$

$$\frac{J_{net(s)}}{J_{tube(s)}} = 1 - (8/3/\pi)(d/D)\{1 + (d/D)^2\}$$

$\sigma_{tresca(s)}$	combined stress in the shaft	[MPa]
$K_{tB(s)}$	concentration factor shaft in bending stress	[]
$\sigma_{B(s)}$	bending stress in the shaft	[MPa]

$K_{tA(s)}$	concentration factor shaft in axial stress	[]
$\sigma_{A(s)}$	axial stress in the shaft	[MPa]
K_{ts}	concentration factor shaft in torsion stress	[]
τ_s	torsion stress in the shaft	[MPa]
$\tau_{s(s)}$	shear stress in the shaft	[MPa]
σ_{Call}	allowable combined stress	[MPa]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]

Concentration factor shaft in bending stress:

$$K_{tB(s)} = 3{,}000 - 6{,}250\left(\frac{d}{D}\right) + 41{,}000\left(\frac{d}{D}\right)^2 - 45{,}000\left(\frac{d}{D}\right)^3$$

$K_{tB(s)}$	concentration factor shaft in bending stress	[]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]

Concentration factor shaft in axial stress:

$0 \leq d/D \leq 0{,}7$

$$K_{tA(s)} = 12{,}806 - 42{,}602\left(\frac{d}{D}\right) + 58{,}333\left(\frac{d}{D}\right)^2$$

$K_{tA(s)}$	concentration factor shaft in axial stress	[]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]

Concentration factor shaft in torsion stress:

$$K_{ts} = 4{,}000 - 6{,}055\left(\frac{d}{D}\right) + 32{,}764\left(\frac{d}{D}\right)^2 - 38{,}330\left(\frac{d}{D}\right)^3$$

K_{ts}	concentration factor shaft in torsion stress	[]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]

Allowable combined stress:

$$\sigma_{Call} = \frac{R_{p0,2T}}{S_F} * C_c$$

σ_{Call}	allowable combined stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient according to load	[]

Combined stress in the hub:

$$\sigma_{tresca(h)} = \sqrt{\left(\frac{Z_{net(h)}}{Z_{tube(h)}} * K_{tB(h)} * \sigma_{B(h)}\right)^2 + \left(\frac{A_{net(h)}}{A_{tube(h)}} * K_{tA(h)} * \sigma_{A(h)}\right)^2 + 4\left(\left(\frac{J_{net(h)}}{J_{tube(h)}} * K_{th} * \tau_h\right)^2 + \tau_{s(h)}^2\right)} \leq \sigma_{Call}$$

$$\frac{Z_{net(h)}}{Z_{tube(h)}} = 1 - \frac{(16/3/\pi)(d/D_h)[1-(D/D_h)^3]}{1-(D/D_h)^4}$$

$$\frac{A_{net(h)}}{A_{tube(h)}} = 1 - \frac{4/\pi(d/D_h)[1-(D/D_h)]}{1-(D/D_h)^2}$$

$$\frac{J_{net(h)}}{J_{tube(h)}} = 1 - \frac{(8/3/\pi)(d/D_h)\{[1-(D/D_h)^3] + (d/D_h)^2[1-(D/D_h)]\}}{1-(D/D_h)^4}$$

$\sigma_{tresca(h)}$	combined stress in the hub	[MPa]
$K_{tB(h)}$	concentration factor hub in bending stress	[]
$\sigma_{B(h)}$	bending stress in the hub	[MPa]
$K_{tA(h)}$	concentration factor hub in axial stress	[]
$\sigma_{A(h)}$	axial stress in the hub	[MPa]
K_{th}	concentration factor hub in torsion stress	[]
τ_h	torsion stress in the hub	[MPa]
$\tau_{s(h)}$	shear stress in the hub	[MPa]
σ_{Call}	allowable combined stress	[MPa]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]
D_h	diameter of the hub	[mm]

Concentration factor hub in bending stress:

$d/D_h \leq 0{,}4; D/D_h \leq 0{,}9$

$$K_{tB(h)} = C_{1B(h)} + C_{2B(h)}\left(\frac{d}{D_h}\right) + C_{3B(h)}\left(\frac{d}{D_h}\right)^2 + C_{4B(h)}\left(\frac{d}{D_h}\right)^3$$

$$C_{1B(h)} = 3{,}000$$

$$C_{2B(h)} = -6{,}250 - 0{,}585(D/D_h) + 3{,}115(D/D_h)^2$$

$$C_{3B(h)} = 41{,}000 - 1{,}071(D/D_h) - 6{,}746(D/D_h)^2$$

$$C_{4B(h)} = -45{,}000 + 1{,}389(D/D_h) + 13{,}889(D/D_h)^2$$

$K_{tB(h)}$	concentration factor hub in bending stress	[]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]

D_h	diameter of the hub	[mm]
$C_{1B(h)}$	coefficient	[]
$C_{2B(h)}$	coefficient	[]
$C_{3B(h)}$	coefficient	[]
$C_{4B(h)}$	coefficient	[]

Concentration factor hub in axial stress:

$0 < D/D_h \leq 0{,}9; d/D_h \leq 0{,}45$

$$K_{tA(h)} = C_{1A(h)} + C_{2A(h)}\left(\frac{d}{D_h}\right) + C_{3A(h)}\left(\frac{d}{D_h}\right)^2$$

$$C_{1A(h)} = 3{,}000$$

$$C_{2A(h)} = 0{,}427 - 6{,}770(D/D_h) + 22{,}698(D/D_h)^2 - 16{,}670(D/D_h)^3$$

$$C_{3A(h)} = 11{,}357 + 15{,}665(D/D_h) - 60{,}929(D/D_h)^2 + 41{,}501(D/D_h)^3$$

$K_{tA(h)}$	concentration factor hub in axial stress	[]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]
D_h	diameter of the hub	[mm]
$C_{1A(h)}$	coefficient	[]
$C_{2A(h)}$	coefficient	[]
$C_{3A(h)}$	coefficient	[]

Concentration factor hub in torsion stress:

$d/D_h \leq 0{,}4; D/D_h \leq 0{,}8$

$$K_{th} = C_{1(h)} + C_{2(h)}\left(\frac{d}{D_h}\right) + C_{3(h)}\left(\frac{d}{D_h}\right)^2 + C_{4(h)}\left(\frac{d}{D_h}\right)^3$$

$$C_{1(h)} = 4{,}000$$

$$C_{2(h)} = -6{,}055 + 3{,}184(D/D_h) - 3{,}461(D/D_h)^2$$

$$C_{3(h)} = 32{,}764 - 30{,}121(D/D_h) + 39{,}887(D/D_h)^2$$

$$C_{4(h)} = -38{,}330 + 51{,}542\sqrt{D/D_h} - 27{,}483(D/D_h)$$

K_{th}	concentration factor hub in torsion stress	[]
D	diameter of the shaft	[mm]
d	diameter of the pin	[mm]
D_h	diameter of the hub	[mm]
$C_{1(h)}$	coefficient	[]
$C_{2(h)}$	coefficient	[]

$C_{3(h)}$	coefficient	[]
$C_{4(h)}$	coefficient	[]

Literature:

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