Square head for shaft-hub connection

The advantage of this connection is easy assembly and disassembly. The disadvantage is the low manufacturing precision and consequent consequences for limited speeds and small torques.
For a simplified calculation, it is assumed that the joint is without will, and that the torque causes the contact stress to be half of each function area of the square head. It is possible to assume a triangular distribution of this stress.
Load distribution will differ from the assumption due to production inaccuracy due to looseness or prestressing of joints and shaft deformations by from torsion torque. These deviations can include in the calculation a coefficient max. stress $S_s = 1{,}3 - 2$ the lower value of which applies to short joints $l \leq s$ and for high accuracy of manufacturing.

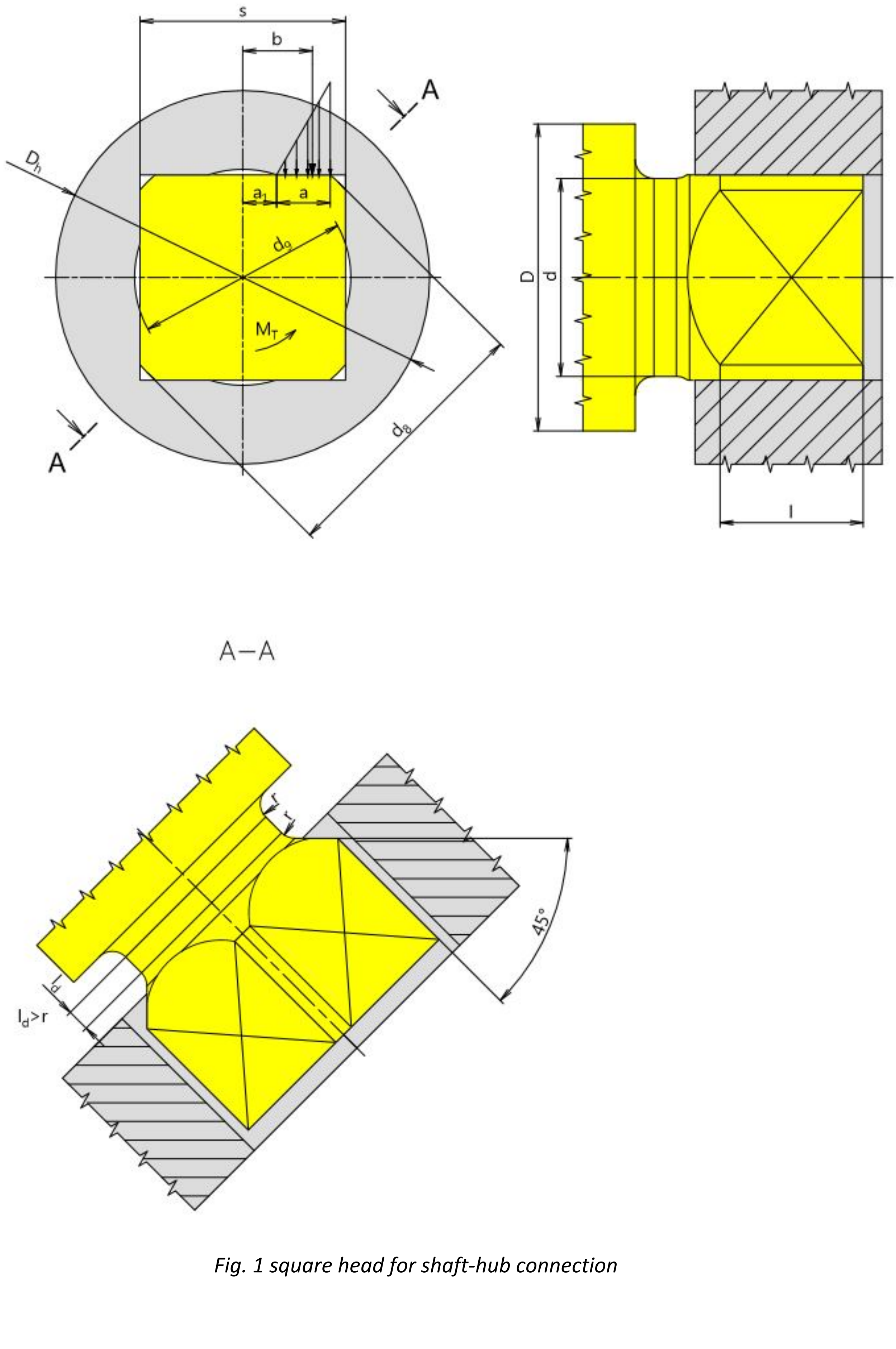

Fig. 1 square head for shaft-hub connection

Bearing stress:

$$p = \frac{M_T * s_s}{2a * l * b} \leq \sigma_{all}$$

p	bearing stress	[MPa]
M_T	torque	[Nm]
s_s	coefficient of maximum stress increase	[]
a	length square head load	[mm]
l	length square head in the hub	[mm]
b	distance of the resultant of the pressure	[mm]

Distance of the resultant of the pressure:

$$b = a_1 + \frac{2}{3}a$$

b	distance of the resultant of the pressure	[mm]
a_1	length square head without load	[mm]
a	length square head load	[mm]

Length square head without load:

$$a_1 = \frac{d_9}{2}\sin\left(\cos^{-1}\frac{s}{d_9}\right)$$

a_1	length square head without load	[mm]
d_9	free diameter	[mm]
s	width square head	[mm]

Length square head with load:

$$a = \frac{d_8}{2}\sin\left(\cos^{-1}\frac{s}{d_8}\right) - a_1$$

a	length square head with load	[mm]
d_8	diameter square head	[mm]
s	width square head	[mm]
a_1	length square head without load	[mm]

Allowable bearing stress:

$$\sigma_{all} = \frac{0{,}9R_{p0,2T}}{S_F} * C_c$$

σ_{all}	allowable bearing stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient of use of joints according to load	[]

Coefficient of use of joints according to load:

load	[]
Unidirectional load, non-impact load	0,8
Unidirectional load, with a small impact load	0,7
Unidirectional load, with a big impact load	0,6
Alternating load, with a small impact load	0,45
Alternating load, with a big impact load	0,25

Torsion stress in the shaft:

$$\tau_s = \frac{16M_T}{\pi d^3} \le \tau_{all}$$

τ_s torsion stress in the shaft [MPa]
M_T torque [Nm]
d diameter of the shaft [mm]
τ_{all} allowable shear stress [MPa]

Allowable shear stress:

$$\tau_{all} = \frac{0{,}4R_{p0,2T}}{S_F} * C_c$$

τ_{all} allowable shear stress [MPa]
$R_{p0,2T}$ the minimum yield strength or 0,2% proof strength at calculation temperature [MPa]
S_F safety factor []
C_c coefficient of use of joints according to load []

Torsion stress in the hub:

$$\tau_h = 3{,}962 \frac{16M_T}{\pi\left((D_h^4 - 4s^4)/D_h\right)} \le \tau_{all}$$

τ_h torsion stress in the hub [MPa]
M_T torque [Nm]
D_h diameter of the hub [mm]
s width square head [mm]
τ_{all} allowable shear stress [MPa]

If the shaft is loaded with the bending moment in the joint, the bending stress must be checked. If the shaft is loaded with a shear force in the joint, the shear stress must be checked. The shaft may be load in the joint by axial force. The shaft must be checked for axial stresses. When calculating the different load types, it is necessary to calculate the combined stress.

Bending stress in the shaft:

$$\sigma_B = \frac{32M_B}{\pi d^3} \le \sigma_{Ball}$$

σ_B	bending stress in the shaft	[MPa]
M_B	bending moment	[Nm]
d	diameter of the shaft	[mm]
σ_{Ball}	allowable bending stress	[MPa]

Allowable bending stress:

$$\sigma_{Ball} = \frac{0{,}6R_{p0,2T}}{S_F} * C_c$$

σ_{Ball}	allowable bending stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient of use of joints according to load	[]

Shear stress in the shaft:

$$\tau_{s(s)} = \frac{4F_R}{\pi d^2} \leq \tau_{all}$$

$\tau_{s(s)}$	shear stress in the shaft	[MPa]
F_R	shear forces	[N]
d	diameter of the shaft	[mm]
τ_{all}	allowable shear stress	[MPa]

Axial stress in the shaft:

$$\sigma_A = \frac{4F_A}{\pi d^2} \leq \sigma_{Aall}$$

σ_A	axial stress in the shaft	[MPa]
F_A	axial forces	[N]
d	diameter of the shaft	[mm]
σ_{Aall}	allowable axial stress	[MPa]

Allowable axial stress:

$$\sigma_{Aall} = \frac{0{,}45R_{p0,2T}}{S_F} * C_c$$

σ_{Aall}	allowable axial stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient of use of joints according to load	[]

Combined stress in the shaft:

$$\sigma_{tresca} = \sqrt{(K_{tB} * \sigma_B)^2 + (K_{tA} * \sigma_A)^2 + 4\left((K_{ts} * \tau_s)^2 + \tau_{s(s)}^2\right)} \leq \sigma_{Call}$$

σ_{tresca}	combined stress in the shaft	[MPa]
K_{tB}	concentration factor in bending	[]

σ_B	bending stress in the shaft	[MPa]
K_{tA}	concentration factor in axial	[]
σ_A	axial stress in the shaft	[MPa]
K_{ts}	concentration factor in torsion	[]
τ_s	torsion stress in the shaft	[MPa]
$\tau_{s(s)}$	shear stress in the shaft	[MPa]
σ_{Call}	allowable combined stress	[MPa]

Concentration factor in bending:

$$K_{tB} = C_{1B} + C_{2B}\left(\frac{D-d}{D}\right) + C_{3B}\left(\frac{D-d}{D}\right)^2 + C_{4B}\left(\frac{D-d}{D}\right)^3$$

$$C_{1B} = 0{,}947 + 1{,}206\sqrt{\frac{D-d}{2r}} - 0{,}131\frac{D-d}{2r}$$

$$C_{2B} = 0{,}022 - 3{,}405\sqrt{\frac{D-d}{2r}} + 0{,}915\frac{D-d}{2r}$$

$$C_{3B} = 0{,}869 + 1{,}777\sqrt{\frac{D-d}{2r}} - 0{,}555\frac{D-d}{2r}$$

$$C_{4B} = -0{,}810 + 0{,}422\sqrt{\frac{D-d}{2r}} - 0{,}260\frac{D-d}{2r}$$

K_{tB}	concentration factor in bending	[]
C_{1B}	coefficient	[]
C_{2B}	coefficient	[]
C_{3B}	coefficient	[]
C_{4B}	coefficient	[]
D	diameter of the shaft	[mm]
d	diameter of the shaft	[mm]
r	radius	[mm]

Concentration factor in axial:

$$K_{tA} = C_{1A} + C_{2A}\left(\frac{D-d}{D}\right) + C_{3A}\left(\frac{D-d}{D}\right)^2 + C_{4A}\left(\frac{D-d}{D}\right)^3$$

$$C_{1A} = 0{,}926 + 1{,}157\sqrt{\frac{D-d}{2r}} - 0{,}099\frac{D-d}{2r}$$

$$C_{2A} = 0{,}012 - 3{,}036\sqrt{\frac{D-d}{2r}} + 0{,}961\frac{D-d}{2r}$$

$$C_{3A} = -0{,}302 + 3{,}977\sqrt{\frac{D-d}{2r}} - 1{,}744\frac{D-d}{2r}$$

$$C_{4A} = 0{,}365 - 2{,}098\sqrt{\frac{D-d}{2r}} + 0{,}878\frac{D-d}{2r}$$

K_{tA}	concentration factor in axial	[]
C_{1A}	coefficient	[]
C_{2A}	coefficient	[]
C_{3A}	coefficient	[]
C_{4A}	coefficient	[]
D	diameter of the shaft	[mm]
d	diameter of the shaft	[mm]
r	radius	[mm]

Concentration factor in torsion:

$$K_{ts} = C_{1s} + C_{2s}\left(\frac{D-d}{D}\right) + C_{3s}\left(\frac{D-d}{D}\right)^2 + C_{4s}\left(\frac{D-d}{D}\right)^3$$

$$C_{1s} = 0{,}905 + 0{,}783\sqrt{\frac{D-d}{2r}} - 0{,}075\frac{D-d}{2r}$$

$$C_{2s} = -0{,}437 - 1{,}969\sqrt{\frac{D-d}{2r}} + 0{,}553\frac{D-d}{2r}$$

$$C_{3s} = 1{,}557 + 1{,}073\sqrt{\frac{D-d}{2r}} - 0{,}578\frac{D-d}{2r}$$

$$C_{4s} = -1{,}061 + 0{,}171\sqrt{\frac{D-d}{2r}} + 0{,}086\frac{D-d}{2r}$$

K_{ts}	concentration factor in torsion	[]
C_{1s}	coefficient	[]
C_{2s}	coefficient	[]
C_{3s}	coefficient	[]
C_{4s}	coefficient	[]
D	diameter of the shaft	[mm]
d	diameter of the shaft	[mm]

r	radius	[mm]

Allowable combined stress:

$$\sigma_{Call} = \frac{R_{p0,2T}}{S_F} * C_c$$

σ_{Call}	allowable combined stress	[MPa]
$R_{p0,2T}$	the minimum yield strength or 0,2% proof strength at calculation temperature	[MPa]
S_F	safety factor	[]
C_c	coefficient according to load	[]

Literature:

AISC: Specification for structural steel buildings: Allowable Stress design and plastic design 1989
Walter D. Pilkey, Deborah F. Pilkey: Peterson’s stress concentration factors. 2008
Joseph E. Shigley, Charles R. Mischke, Richard G. Budynas: Konstruování strojních součástí 2010.
MET-Calc: Allowable stress.
https://met-calc.com/soubory/clanky/Allowable%20stress%20[EN].pdf
A. Bolek, J. Kochman a kol.: Části a mechanismy strojů I. 1989.
K. Kříž a kol.: Strojní součásti 1. 1984.